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Asymptotics of the Modules of Mirror Symmetric Doubly Connected Domains under Stretching

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Abstract—An asymptotic formula for the conformal module of a doubly connected domain symmetric about the abscissa axis under an unbounded stretching in the direction of this axis is obtained. Thereby, in the case of symmetric domains, a question asked by Vuorinen is answered.

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1. INTRODUCTION

Consider a doubly connected plane domain D with nondegenerate boundary components. One of its most important characteristics is the conformal module $m(D)$ (see, e.g., [1]). By definition, if D is conformally equivalent to an annulus $\{r_1 < |z| < r_2\}$, then

$$m(D) := \frac{1}{2\pi} \ln \frac{r_2}{r_1}.$$

The module can also be calculated by using the concept of extremal length (see, e.g., [1]; [2, Chap. 1]). Thus, $m(D) = \lambda(\Gamma)$, where $\lambda(\Gamma)$ is the extremal length of the family of curves Γ in the domain D which join the boundary components of D. Moreover, $m(D) = 1/\lambda(\Gamma')$, where Γ' is the family of curves separating the boundary components in D. Finally, $m(D) = 1/\operatorname{Cap}(C)$, where $\operatorname{Cap}(C)$ is the conformal capacity of a condenser whose field is the domain D and plates are the boundary components of D.

During the past years, the conformal modules of doubly connected domains, as well as the related modules of quadrilaterals (see Sec. 2), have been extensively studied both in the case of polygonal and arbitrary boundaries (see, e.g., [1], [3]–[10]).

The module of a domain is invariant under conformal mappings and quasi-invariant under quasiconformal mappings: if f is an H-quasiconformal mapping of a domain D onto $\widetilde{D}$, then

$$\frac{1}{H} m(D) \le m(\widetilde{D}) \le H m(D)$$

(see, e.g., [1]; [2, Chap. 1]).

One of the simplest H-quasiconformal mappings is a stretching along the abscissa axis, that is,

$$f_H \colon x + iy \mapsto Hx + iy, \tag{1.1}$$

where $H > 1$. Vuorinen posed the problem of investigating the distortion of the conformal module of a domain D under the action of f_H, in particular, its asymptotic behavior as $H \to \infty$ (see [8]). We have previously studied the cases where D is the difference of two homothetic squares with sides parallel to the coordinate axes [8] (a rectangular frame) or of those with vertices on the coordinate axes [11] (a

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